MODERN TECHNIQUES IN COMBINATORIAL SET THEORY PRESENTATION MATERIAL

Topic 1 - Inductive constructions and closure arguments

Any of the following papers/books will naturally continue the topics covered in the course:

- Set theory and Ramsey theory in Euclidean spaces [14, 7, 9, 11];
- Diagonalisations of length continuum and general topology [28, Chapter 4];
- Soifer's colouring book [24];
- Some chromatic number papers [15, 10, 17];
- Erdős' work in infinite combinatorics [16];
- Elementary submodels in combinatorics [25, 26];
- Elementary submodels in topology [6, 8];
- Balogh's work with elementary submodels and topology [2, 3, 1];
- From the theory of countably infinite graphs [5], [4, Chapter 8];
- Descriptive graph combinatorics [13].

Topic 2 - Coherent sequences and minimal walks

The main reference here is S. Todorcevic's [29] and recommended additional topics from the book include:

- Banach spaces with few operators (Chapter 5.3);
- Variations of the square-bracket relation (Chapter 2.3 and 5.1);
- Coherent mappings in general (Chapter 4).

A great survey on combinatorial topics is Todorcevic's [27] focusing on graph dichotomies and compactness results.

J. Moore's solution to the basis problem for uncountable linear orders [22] and the structural analysis of Aronszajn trees [20] is also recommended.

Topic 3 - Construction schemes

- Trees of elementary submodels (Davies-trees) [25];
- The Steinhaus tiling problem [12];
- Todorcevic's construction scheme [19, 18];
- Kurepa families and Jensen matrices [29, Chapter 7.6];
- Constructions based on diamond [23, 21].

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