MAT135H1F – Quiz 3

TUT5101

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FAMILY NAME: .... GIVEN NAME: ....

Mark your lecture and tutorial sections: STUDENT ID: ....

L0101	L5101	T0101	T0201	T5101	T5102

You have 25 minutes to solve the problems below! Each problem is worth 1 point. Good luck!

Question 1. *What is the slope of the tangent to* $f(x) = \arccos(2x-1))$ *at* $x = \frac{1}{2}$*?*

(a) -2

(b) 2

(c) -1

(d) 1

Answer: (a) -2. First we determine $f'(x)$. Using the chain rule and our knowledge of differentiating arccos, we obtain $f'(x) = -2\frac{1}{\sqrt{1-(2x-1)^2}} = \frac{-2}{\sqrt{4x-4x^2}} = \frac{-1}{\sqrt{x(1-x)}}$. The slope of the line tangent to f at $x = \frac{1}{2}$ is given by $f'(\frac{1}{2}) = \frac{-1}{\sqrt{\frac{1}{4}}} = -2$.

Question 2. *Let* $g(0) = 0$ *and* $g'(0) = 4$*. What is* $\frac{d}{dx}g(g(x))$ *at* $x = 0$*?*

(a) 4

(b) 0

(c) 8

(d) 16

Answer: (d) 16. Differentiating $g(g(x))$ with the chain rule gives $\frac{d}{dx}g(g(x)) = g'(g(x))g'(x)$. Thus, evaluating at $x = 0$ gives $g'(g(0))g'(0) = g'(0)^2 = 16$.

Question 3. *Let* $x^2 + xy + y^2 = 4$*. Find* y' *by implicit differentiation.*

(a) $-\frac{2x+y}{y+x}$

(b) $-\frac{x+2y}{y+2x}$

(c) $-\frac{2x+y}{2y+x}$

(d) $-\frac{3y+2x}{x}$

Answer: (c) $-\frac{2x+y}{2y+x}$. First we differentiate both sides of the equation using the chain rule and the product rule on the left side. We obtain $2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$. This implies $\frac{dy}{dx}(x+2y) = -2x - y$ which gives $\frac{dy}{dx} = \frac{-2x-y}{2y+x}$.

Question 4. *Where is the horizontal tangent of* $f(x) = e^{x^2}$*?*

(a) At $x = 0$

(b) At $x = -1$

(c) There is none.

(d) At $x = 1$

Answer: (a) at $x = 0$. We solve for $f'(x)$ using the chain rule. We have $f'(x) = 2xe^{x^2}$. Horizontal lines have a slope of 0, so we solve $f'(x) = 0$. Since $e^{x^2} > 0$ for all $x \in \mathbf{R}$, we must have $x = 0$.

Question 5. *What is* $\frac{d}{dx}\ln^2(x)$ *at* $x = e$*?*

(a) 1

(b) 2

(c) $2\ln(e)$

(d) $\frac{2}{e}$

Answer: (d) $\frac{2}{e}$. We differentiate $\ln^2(x)$ using the chain rule. We have $\dfrac{d}{dx}\ln^2(x) = 2\ln(x)\frac{1}{x}$. At $x = e$, we obtain $2\ln(e)\frac{1}{e} = 2e^{-1}$.

Question 6. *What is* $\frac{d}{dx}\sin(\frac{\pi}{x})$ *at* $x = 1$*?*

(a) -1

(b) 1

(c) $-\pi$

(d) π

Answer: (c) $-\pi$. We differentiate using the chain rule. We have $\dfrac{d}{dx}\sin(\frac{\pi}{x}) = \cos(\frac{\pi}{x})\frac{-\pi}{x^2}$. Evaluating at $x = 1$ gives $\cos(\pi)(-\pi) = -\pi$.