MAT135H1F – Quiz 2

TUT0101

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FAMILY NAME: ..... GIVEN NAME: .....

Mark your lecture and tutorial sections: STUDENT ID: .....

L0101	L5101	T0101	T0201	T5101	T5102

You have 25 minutes to solve the problems below! Each problem is worth 1 point. Good luck!

Question 1. *What is* $\lim_{x\to 0} \frac{\sin(x)}{2x}$ *?*

(a) $\frac{1}{2}$

(b) 0

(c) Does not exist.

(d) π

Answer: (a) $\frac{1}{2}$. Pulling out a factor of $\frac{1}{2}$ gives $\lim_{x\to 0} \frac{\sin(x)}{2x} = \frac{1}{2}\lim_{x\to 0} \frac{\sin(x)}{x} = \frac{1}{2}$.

Question 2. *Let* $f(x) = \frac{e^x}{x^3}$. *What is* $f'(2)$*?*

(a) $-\frac{e^2}{16}$

(b) 0

(c) $\frac{e^2}{8}$

(d) $-\frac{\ln 2}{16}$

Answer: (a) $-\frac{e^2}{16}$. Differentiating $f(x)$ with the quotient rule gives $f'(x) = \frac{x^3e^x - 3x^2e^x}{x^6} = \frac{e^x(x-3)}{x^4}$ so $f'(2) = -\frac{e^2}{16}$.

Question 3. *Let* $f(x) = \frac{1}{2}e^x - 3x$. *The equation of the tangent line to the graph of* $f(x)$ *at* $x = \ln 6$ *is:*

(a) $y = e^6x + 3$

(b) $y = -x + 3$

(c) $y = 0$

(d) $y = 3(1 - \ln 6)$

Answer: (d) $y = 3(1 - \ln 6)$. We start by computing $f'(\ln 6)$ which is the slope of the desired tangent line. We have $f'(x) = \frac{1}{2}e^x - 3$ so $f'(\ln 6) = 0$. Thus the tangent line is horizontal with equation $y = b$ for some constant b. We know $(\ln 6, f(\ln 6))$ is a point on the line so we conclude $y = f(\ln 6) = \frac{1}{2}e^{\ln 6} - 3\ln 6 = 3(1 - \ln 6)$.

Question 4. *What is the 8th derivative of* $x + x^6 - x^7$*?*

(a) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

(b) 1

(c) 0

(d) None of the above.

Answer: (c) 0. We know that the derivative of a sum is equal to the sum of derivatives. This means that we may consider each term of $x + x^6 - x^7$ independently. Differentiating each monomial term once reduces the power of the monomial by one. Thus, differentiating n times a monomial x^n reduces the power by n and

leaves us with a constant. Then, differentiating once more gives 0. Namely, differentiating a monomial more times than the degree gives 0. For example $\frac{d^7}{dx^7}x^7 = 7!$ so $\frac{d^8}{dx^8}x^7 = 0$. Therefore, we conclude that $\frac{d^8}{dx^8}(x + x^6 - x^7) = 0$.

Question 5. *What is* $\frac{d}{dx}\cos^2(x)$ *at* $x = \pi/4$*?*

(a) 0

(b) 1

(c) -1

(d) $\frac{1}{\sqrt{2}}$

Answer: (c) -1. We differentiate using the chain rule to obtain $\frac{d}{dx}\cos^2(x) = 2\cos(x)(-\sin(x)) = -2\cos(x)\sin(x)$. Thus, evaluating at $\frac{\pi}{4}$ gives $-2\cos(\frac{\pi}{4})\sin(\frac{\pi}{4}) = -1$.

Question 6. *Sketch the graph of an everywhere continuous function that is not differentiable at* $x = 2$.

Answer: A sharp point on a graph is an example of an instance where the left and right limits exist and and equal, the function is defined but there is no derivative. We can draw a graph with a sharp point at $x = 2$.