MAT136H1F – Quiz 6

TUT0101 – M3 (TA: I. Angelopoulos)

Fall, 2014

FAMILY NAME: .... GIVEN NAME: ....

STUDENT ID: ....

Mark your lecture and tutorial sections:

L0101 (morning)	L5101 (evening)	T0101 (M3)	T0102 (R4)	T5101 (T5)	T5201 (R5)

You have 15 minutes to solve the problems. Each problem is worth 2 points. Good luck!

Question 1. *Find a power series representation of* $\int \frac{1}{1+x^3}dx$.

$$\frac{1}{1+x^3} = \frac{1}{1-(-x^3)} = 1 - x^3 + x^6 - x^9 + x^{12} - x^{15} + \cdots \qquad |-x^3| < 1 \iff |x| < 1$$

$$\int \frac{dx}{1+x^3} = C + x - \frac{x^4}{4} + \frac{x^7}{7} - \frac{x^{10}}{10} + \frac{x^{13}}{13} - \frac{x^{16}}{16} + \cdots \qquad |x| < 1$$

It's probably better to use sigma-notation, but I decided to just write out the first few terms of the series.

Question 2. *Find the interval of convergence for* $\sum_{n=1}^{\infty} \frac{2^n (x-2)^n}{n}$.

Ratio Test: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \left[\frac{2^{n+1}(x-2)^{n+1}}{(n+1)}\right] / \left[\frac{2^n(x-2)^n}{n}\right] \right| = \lim_{n\to\infty} \left| 2(x-2)\left(\frac{n}{n+1}\right) \right|$

$= 2|x-2|$

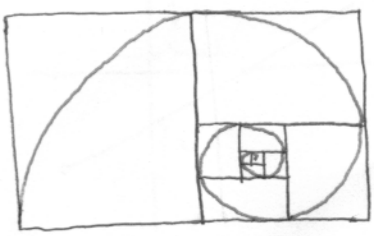
So if $2|x-2| < 1$ i.e. $|x-2| < \frac{1}{2}$ $\sum_{n=1}^{\infty} \frac{2^n(x-2)^n}{n}$ converges

and if $2|x-2| > 1$ the series diverges

check endpoints $x = \frac{5}{2}$: $\sum_{n=1}^{\infty} \frac{2^n\left(\frac{1}{2}\right)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$ divergent $\quad x = \frac{3}{2}$: $\sum_{n=1}^{\infty} \frac{2^n\left(-\frac{1}{2}\right)^n}{n} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$ Converges

$\left[\frac{3}{2}, \frac{5}{2}\right)$ interval of convergence

Question 3. *Congratulations, this was the last quiz and you just won 2 free points! Draw something nice for your TA or just leave this place blank.* ***Don't forget to complete the course evaluation on Portal.***



MAT136H1F – Quiz 6

TUT0201 – R4 (TA: B. Navarro Lameda)

Fall, 2014

FAMILY NAME: .... GIVEN NAME: ....

STUDENT ID: ....

Mark your lecture and tutorial sections:

L0101 (morning)	L5101 (evening)	T0101 (M3)	T0102 (R4)	T5101 (T5)	T5201 (R5)

You have 15 minutes to solve the problems. Each problem is worth 2 points. Good luck!

Question 1. *Find a power series representation of* $\int \frac{1}{2-3x}dx$.

$$\frac{1}{2-3x} = \frac{1}{2}\left(\frac{1}{1-\frac{3x}{2}}\right) = \frac{1}{2}\left(1+\frac{3x}{2}+\frac{9x^2}{4}+\frac{27}{8}x^3+\cdots\right) \qquad \left|\frac{-3x}{2}\right|<1 \iff |x|<\frac{2}{3}$$

$$\int\frac{dx}{2-3x} = C + \frac{1}{2}\left(x+\frac{3}{4}x^2+\frac{9}{12}x^3+\frac{27}{32}x^4+\cdots\right) \qquad |x|<\frac{2}{3}$$

$$= C + \frac{x}{2} + \frac{3}{8}x^2 + \frac{9}{24}x^3 + \frac{27}{64}x^4 + \cdots \qquad |x|<\frac{2}{3}$$

It's probably better to use sigma-notation, but I decided to just write out the first few terms in the series.

Question 2. *Find the radius of convergence for* $\sum_{n=1}^{\infty}\frac{(x+2)^n}{\sqrt{n!}}$.

Ratio Test: $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\left[\frac{(x+2)^{n+1}}{\sqrt{(n+1)!}}\right]/\left[\frac{(x+2)^n}{\sqrt{n!}}\right]\right| = \lim_{n\to\infty}\left|(x+2)\sqrt{\frac{n!}{(n+1)!}}\right|$

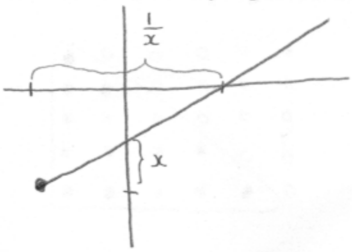
$= \lim_{n\to\infty}\left|(x+2)\sqrt{\frac{1}{n+1}}\right|$

$= 0$ (no matter what x is!)

Therefore $\sum_{n=1}^{\infty}\frac{(x+2)^n}{\sqrt{n!}}$ converges for any x.

So the radius of convergence is ∞.

Question 3. *Congratulations, this was the last quiz and you just won 2 free points! Draw something nice for your TA or just leave this place blank.* ***Don't forget to complete the course evaluation on Portal.***



MAT136H1F – Quiz 6

TUT5101 – T5 (TA: A. Stewart)

Fall, 2014

FAMILY NAME: .... GIVEN NAME: ....

STUDENT ID: ....

Mark your lecture and tutorial sections:

L0101 (morning)	L5101 (evening)	T0101 (M3)	T0102 (R4)	T5101 (T5)	T5201 (R5)

You have 15 minutes to solve the problems. Each problem is worth 2 points. Good luck!

Question 1. *Find the interval of convergence for* $\sum_{n=1}^{\infty} \frac{(x-1)^n}{2n+1}$.

Ratio Test: $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\left[\frac{(x-1)^{n+1}}{2(n+1)+1}\right] / \left[\frac{(x-1)^n}{2n+1}\right]\right| = \lim_{n\to\infty}\left|(x-1)\left(\frac{2n+1}{2n+3}\right)\right| = |x-1|$

So if $|x-1|<1$, $\sum_{n=1}^{\infty}\frac{(x-1)^n}{2n+1}$ converges. If $|x-1|>1$ $\sum_{n=1}^{\infty}\frac{(x-1)^n}{2n+1}$ diverges.

Therefore the radius of convergence is 1.

Interval of Convergence: $[0,2)$

Check endpoints

$x=2$ $\sum_{n=1}^{\infty}\frac{1}{2n+1} \geqslant \sum_{n=1}^{\infty}\frac{1}{2n+2} = \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n+1}$ DIVE[RGENT] GE[...]

$x=0$ $\sum_{n=1}^{\infty}(-1)^n\frac{1}{2n+1}$ CONVERGENT (Alt. Series test)

Question 2. *Find a power series representation of* $\ln(1-2x)$.

$f(x) = \ln(1-2x)$

$f'(x) = \frac{-2}{1-2x} = -2\left(\frac{1}{1-2x}\right) = -2\left(1+2x+4x^2+8x^3+\cdots\right) = -2-4x-8x^2-16x^3-\cdots \quad |2x|<1$

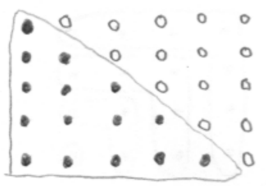
$|x|<\frac{1}{2}$

$\int\frac{-2}{1-2x}dx = C - 2x - 2x^2 - \frac{8}{3}x^3 - 4x^4 - \frac{32}{5}x^5 - \cdots \quad |x|<\frac{1}{2}$

$\ln(1-2(0)) = \ln(1) = 0 = C$

Therefore $\ln(1-2x) = -2x - 2x^2 - \frac{8}{3}x^3 - 4x^4 - \frac{32}{5}x^5 - \cdots \quad |x|<\frac{1}{2}$

Question 3. *Congratulations, this was the last quiz and you just won 2 free points! Draw something nice for your TA or just leave this place blank.* **Don't forget to complete the course evaluation on Portal.**



MAT136H1F – Quiz 6

TUT5201 – R5 (TA: B. Navarro Lameda)

Fall, 2014

FAMILY NAME: .... GIVEN NAME: ....

STUDENT ID: ....

Mark your lecture and tutorial sections:

L0101 (morning)	L5101 (evening)	T0101 (M3)	T0102 (R4)	T5101 (T5)	T5201 (R5)

You have 15 minutes to solve the problems. Each problem is worth 2 points. Good luck!

Question 1. *Find the radius of convergence for* $\sum_{n=1}^{\infty} \frac{3^n x^n}{n^5}$.

Ratio Test: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \left[\frac{3^{n+1}x^{n+1}}{(n+1)^5}\right] \Big/ \left[\frac{3^n x^n}{n^5}\right] \right| = \lim_{n\to\infty} \left| 3x \left(\frac{n}{n+1}\right)^5 \right| = 3|x|$

So if $3|x|<1$, i.e. $|x|<\frac{1}{3}$, $\sum_{n=1}^{\infty} \frac{3^n x^n}{n^5}$ converges. If $3|x|>1$, $\sum_{n=1}^{\infty} \frac{3^n x^n}{n^5}$ diverges.

Therefore the radius of convergence is $1/3$.

Question 2. *Find a power series representation of* $\int \frac{1}{2+x^5}dx$.

$\frac{1}{2+x^5} = \frac{1}{2}\left(\frac{1}{1+\frac{x^5}{2}}\right) = \frac{1}{2}\left(1 - \frac{x^5}{2} + \frac{x^{10}}{4} - \frac{x^{15}}{8} + \cdots\right)$ valid if $\left|\frac{x^5}{2}\right|<1 \Leftrightarrow |x|<\sqrt[5]{2}$

Term-by-term integration says:

$\int \frac{dx}{2+x^5} = C + \frac{1}{2}\left(x - \frac{x^6}{12} + \frac{x^{11}}{44} - \frac{x^{16}}{16(8)} + \cdots\right) = C + \frac{x}{2} - \frac{x^6}{24} + \frac{x^{11}}{88} - \cdots \quad |x|<\sqrt[5]{2}$

It's probably better to use sigma-notation, but I decided to write out the series to illustrate.

Question 3. *Congratulations, this was the last quiz and you just won 2 free points! Draw something nice for your TA or just leave this place blank.* ***Don't forget to complete the course evaluation on Portal.***