PROBLEM SET 9

DUE DATE: JUNE 12, 2019

Exercise 9.1. *Suppose that L is a linear order so that L and $-L$ has no common infinite suborder. Prove that either L or $-L$ is well ordered.*

Exercise 9.2. *Prove that a Countryman line L cannot be ccc i.e., there must be an uncountable collection of pairwise disjoint non-empty intervals in L.*

Problem 9.3. *Show that for any cardinal κ, there is a linear order of size κ which has more than κ initial segments.*

Problem 9.4. *Use the Continuum Hypothesis e.g., $2^{\aleph_0} = \aleph_1$ to construct a universal linear order L of size $\aleph_1$ inside $(\omega^\omega, <^*)$. That is, L has size $\aleph_1$ and embeds any linear order of size $\aleph_1$.*

We use the usual notation from the lectures regarding minimal walks along a $\underline{C}$-sequence on ω_1.

Exercise 9.5. *Prove that for any $\alpha < \beta$, $\max(F(\alpha, \beta) \cap \alpha) = \lambda(\alpha, \beta)$.*

Problem 9.6. *Prove that for any $\zeta < \alpha < \beta$, $F(\zeta, \alpha) \subset F(\zeta, \beta) \cup F(\alpha, \beta)$.*

Finally, some questions about partial orders.

Problem 9.7. *Let P be a partial order and let σP denote the set of well-ordered subsets of P ordered by end-extension. Show that there is no strictly increasing map from σP to P.*

Problem 9.8. *Show that there is a poset of size $\mathfrak{c}$ in which every chain and anti-chain is countable*

Problem 9.9. *Assume that $S, T \subset \omega_1$ so that $S \setminus T$ is stationary. Let σS and σT denote the set of closed subsets of S and T, respectively. Prove that there is no embedding of σS into σT.*

Problem 9.10. *Using the previous problem, show that there are $2^{\aleph_1}$-many pairwise non-embedable partial orders of size $\mathfrak{c}$.*

This is the last problem set, thanks for the great work!