PROBLEM SET 8

DUE DATE: JUNE 5, 2019

Exercise 8.1. *Suppose that P is a partial order and any antichain in P has finite size at most k. Show that P is the union of at most k chains given that all its finite subsets satisfy this property.*

Let L be a *Countryman line* i.e., a linear order which is uncountable but L^2 is the union of countably many chains (partially ordered by the coordinate-wise order). Without knowing whether such things could exist, let's prove a few things.

Exercise 8.2. *Show that L has no uncountable well-ordered or reverse well-ordered subset.*

Problem 8.3. *Show that L has no uncountable suborder isomorphic to a set of reals.*

Problem 8.4. *Show that L and its reverse $-L$ has no common uncountable suborder.*

The *maximal weight* is an important characteristic of minimal walks: define

$$\rho_1 : [\omega_1]^2 \to \omega$$

by

$$\rho_1(\alpha, \beta) = \max\{|C_\xi \cap \alpha| \colon \xi \in Tr(\alpha, \beta)\}.$$

In other words, $\rho_1(\alpha, \beta) = \max \rho_0(\alpha, \beta)$.

Problem 8.5 (Finite-to-one property)**.** *Show that for any $\beta < \omega_1$ and $n < \omega$, the set*

$$\{\alpha < \beta : \rho_1(\alpha, \beta) \leq n\}$$

is finite.

Problem 8.6 (Coherence of max. weight)**.** *Show that for any $\alpha < \beta < \omega_1$, the set*

$$\{\xi < \alpha : \rho_1(\xi, \beta) \neq \rho_1(\xi, \alpha)\}$$

is finite.

The *number of steps* is another characteristic of minimal walks: define

$$\rho_2 : [\omega_1]^2 \to \omega$$

by

$$\rho_2(\alpha, \beta) = |Tr(\alpha, \beta)| - 1.$$

Problem 8.7 (Semi-coherence of number of steps)**.** *Show that for any $\alpha < \beta < \omega_1$,*

$$\sup_{\xi < \alpha} |\rho_2(\xi, \beta) - \rho_2(\xi, \alpha)| < \infty.$$