PROBLEM SET 7

DUE DATE: MAY 22, 2019

Exercise 7.1. *Prove that any set of* $(r-1)(s-1)+1$ *numbers contains a monotonically increasing subsequence of length* r *or a monotonically decreasing subsequence of length* s.

Exercise 7.2. *Show that* ω_1 *with the order topology is not metrizable.*

Exercise 7.3. *Prove that the Sierpinski colouring witnesses* $\omega_1 \not\to [\omega_1]^2_2$.

Exercise 7.4. *Suppose that the colouring* c *witnesses* $\omega_1 \not\to [\omega_1]^2_{\omega_1}$. *Show that for any* $\alpha < \omega_1$, *there is an infinite* $A \subset \omega_1$ *so that* $c \restriction [A]^2$ *is constant* α.

Exercise 7.5. *Prove that any Aronszajn tree has a subtree with the property that each node has uncountably many extensions.*

Problem 7.6. *Prove that there is a subset* T *of* $\{t \subset \mathbb{Q} : \max t \in \mathbb{Q}\}$ *which forms an Aronszajn tree with the end-extension relation. Show that this* T *must be special.*

Problem 7.7. *Use an Aronszajn tree and Sierpinski's idea to witness* $\omega_1 \not\to [\omega_1]^2_3$.

Problem 7.8. *Prove that for any* $c : \omega_1 \times \omega \to k$ *with* k *finite, there are infinite* $A \subset \omega_1, B \subset \omega$ *so that* $c \restriction A \times B$ *is constant.*

Problem 7.9. *Show that if* V *is a* $\mathfrak{c}^+$*-dimensional vector space over* $\mathbb{Q}$ *and* $c : V \to \omega$ *then there is a monochromatic solution to* $x + y = z$ *with* x, y, z *pairwise distinct and non-zero.*

Problem 7.10. *For any colouring* $c : \mathcal{P}(\omega) \to \omega$, *there is a monochromatic, non-trivial solution to* $X \cup Y = Z$.