PROBLEM SET 4

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Exercise 4.1. *Prove that any monotone* $f : \omega_1 \to \mathbb{R}$ *is eventually constant.*

Exercise 4.2. *Suppose that* $X \in H(\theta)$ *and* $S \subset \kappa$ *is stationary for some regular, uncountable* κ*. Then there is an* $M \prec H(\theta)$ *so that* $X \in M$ *and* $M \cap \kappa \in S$*.*

Exercise 4.3. *Suppose that* $<^*$ *is some well order on* ω_1 *of type* ω_1*. Prove that there is a club* $C \subset \omega_1$ *so that* $\alpha < \beta \in C$ *implies that* $\alpha <^* \beta$ *(i.e., the standard well order and* $<^*$ *must agree on* C*).*

Exercise 4.4. *Suppose that* $\mathcal{F}$ *is a d-almost disjoint family of countably infinite sets. Find a colouring* $c : \bigcup \mathcal{F} \to \omega$ *so that* c *assumes all colours on any element* $a \in \mathcal{F}$*.*

Exercise 4.5. *Take some cardinal* κ *and natural number* n*. We define a graph* $Sh_n(\kappa)$ *on vertices* $[\kappa]^n$ *and edges* $\bar{a}\bar{b}$ *where*

$$a_0 < a_1 = b_0 < a_2 = b_1 < \cdots < a_{n-1} = b_{n-2} < b_{n-1}.$$

Show that for any cardinal χ *there is a large enough* κ*, so that* $Sh_n(\kappa)$ *has chromatic number at least* χ*.*

Problem 4.6. *Suppose* $S \subset \omega_1$ *is a set of limit ordinals and that* C_α *is a cofinal subset of* α *of type* ω *for* $\alpha \in S$*. Show that* S *is non-stationary if and only if* $\{C_\alpha : \alpha \in S\}$ *is essentially disjoint.*

Problem 4.7. *Find a 2-almost disjoint family of sets which is not essentially disjoint.*

Problem 4.8. *Find a family* $\mathcal{F}$ *of countably infinite, almost disjoint sets that has uncountable chromatic number i.e., for any* $c : \bigcup \mathcal{F} \to \omega$ *there is some* $a \in \mathcal{F}$ *so that* $c \restriction a$ *is constant.*

The *colouring number* of a graph G, denote by $Col(G)$, is the minimal κ so that $V(G)$ has a well order $<$ such that for any $v \in V$, $\{u < v : uv \in E(G)\}$ has size $< \kappa$.

Problem 4.9. *Prove that* $\chi(G) \leq Col(G)$ *that is, the chromatic number is at most the colouring number.*

Problem 4.10. *Find a graph* G *such that* $\chi(G) < Col(G)$*.*