University of Toronto

Faculty of Arts and Science
Make-up Term Test (June 12 2015)
MAT135H1F - Calculus I (A)
Duration - 2 hours
No Aids Allowed

Family Name: ______________________________

First Name: ______________________________

Student Number: ______________________________

Lecture and Tutorial section:

L0101	L5101	T0101	T0201	T5101	T5102
TR10-1	TR6-9	TR1	TR2	TR5	TR5

This exam contains 10 pages (including this cover page) and 6 problems. Check to see if any pages are missing and ensure that all required information at the top of this page has been filled in.

No aids are permitted on this examination. Examples of illegal aids include, but are not limited to textbooks, notes, calculators, or any electronic device.

Unless otherwise indicated, you are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work!** Write your answers in the space provided. Work scattered over the page without clear ordering will receive very little credit.
- **Justify your answers!** A correct answer without explanation or algebraic work will receive no credit; an incorrect answer supported by correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	15	
2	15	
3	10	
4	10	
5	10	
6	20	
Total:	80	

1. Answer the following questions on limits by selecting the right answer. You don't need to show your work on this problem.

   (a) (3 points) Find $\lim_{x \to -\infty} 2\arctan(x^2)$.

   (a) π
   (b) $\pi/2$
   (c) 0
   (d) ∞

   (b) (3 points) Find $\lim_{y \to -\infty} \frac{\sqrt{y^4 + 7y - 2}}{4y^2 - 2y + 7}$.

   (a) ∞
   (b) $\frac{1}{4}$
   (c) 4
   (d) 0

   (c) (3 points) Find $\lim_{t \to 0} \frac{\tan(2t)}{\sin(3t)}$.

   (a) ∞
   (b) Does not exist.
   (c) $\frac{2}{3}$
   (d) $\frac{3}{2}$

(d) (3 points) Find the vertical asymptotes of $f(t) = \frac{8-t^3}{t^2-4}$.

(a) $t = 2$

(b) $t = -2$

(c) $t = \pm 2$

(d) There are no vertical asymptotes.

(e) (3 points) Find $\lim_{x \to 1} \frac{e^x - e}{x - 1}$. (Hint: write the limit as the derivative of a function $f(x)$.)

(a) -1

(b) e

(c) 0

(d) $\ln(e)$

2. Answer the following questions on derivatives.

(a) (3 points) Find $\dfrac{d}{dx}\arccos(\sqrt{x+1})$.

(b) (3 points) Your rubber duck is floating in your bathtub. The distance of the duck from the bottom of the tub is $d(t) = 30 + \sin(\frac{\pi}{6}t)$ centimetres t seconds after starting your bath. What is the duck's acceleration 1 minute after starting your bath?

(c) (3 points) Find $\dfrac{d}{dx}e^{\sin(\frac{1}{x})}$.

(d) (3 points) What is the 7^{th} derivative of $(x^2 + x + 1)(x^2 - 1)(2x - 3x^2)$?

(e) (3 points) The velocity of a car at time t is given by the function $v(t) = t^2 - 3t + 1$ in kilometres per hour. At what times is the car slowing down?

3. (a) (5 points) Suppose that f is given by the equation

$$f(x) = \begin{cases} 3x & \text{if } x \leq 0, \\ (x^3 - x)\sin(\frac{1}{x}) & \text{if } x > 0 \end{cases}$$

Show that $f(x)$ is continuous at 0.

(b) (5 points) Use the Intermediate Value Theorem to show that the equation $1 + \sin(x) = x^2$ has at least one solution.

4. (10 points) Find the derivative of $f(x) = \sqrt{x^2 + x + 1}$ at $x = 2$ using the definition of the derivative as a limit.

5. (10 points) Suppose that a curve is given by the equation $x^2y^3 + 2x = y - 1$. What is the equation of the tangent line for this curve when $x = 0$?

6. Answer the following questions about derivatives.

   (a) (5 points) Find $\frac{d}{dt}\frac{\cos(t)}{1-\sin(t)}$. Simplify your solution as much as possible.

   (b) (5 points) Suppose that $g'(1) = g(1)$ is not 0. What is the derivative of $\ln(xg(x))$ at $x = 1$?

(c) (5 points) Find the derivative of $(2^x + \tan(x))^2$ at $x = 0$.

(d) (5 points) Find the 999th derivative of e^{-2r} at $r = 0$.