Colouring problems of Erdős and Rado on infinite graphs

Dániel T. Soukup

University of Toronto, March 20 2015

A brief outline

We work with infinite graphs:
countably infinite or uncountably many vertices.

Edge colouring problems:

- a theorem of R. Rado on path decompositions,
- results on **hypergraphs** ✓
- extensions from paths to **powers of paths** ✓
- extensions to **uncountable graphs** ✓

Vertex colouring problems:

- **structural properties** of graphs with **large chromatic number**,
- classical results revisited ✓
- new results on **chromatic number** and **connectivity** ✓

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Theorem (R. Rado, 1978)

If the edges of the graph $K_{\mathbb{N}}$ are coloured with finitely many colours then

*the vertices can be **covered by disjoint monochromatic paths of different colours**.*

Path decompositions

The origins

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Developments on the finite case

General problem (Gyárfás): given an r-edge colouring of K_n is there a cover by (disjoint) monochromatic paths (of different colours)?

Suppose that r **is small**:

1. ("easy") Every 2-edge coloured K_n can be **partitioned into 2** monochromatic paths of different colours.
2. **A. Pokrovskiy, 2013:** Every 3-edge coloured K_n can be **partitioned into 3** monochromatic paths.

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Definition (Rado, 1978)

For a graph $P = (V, E)$, we say that P is a ***path*** iff there is a **well ordering $\prec$ on V** such that any two points $v, w \in V$ are connected by a $\prec$-monotone finite path.

How to imagine paths **longer than type ω**?

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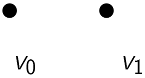

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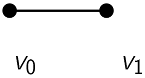

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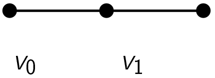

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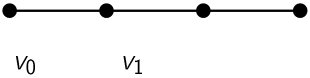

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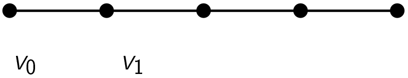

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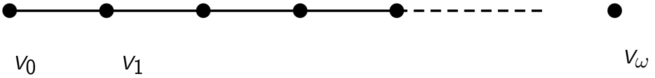

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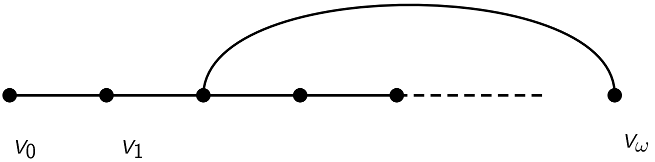

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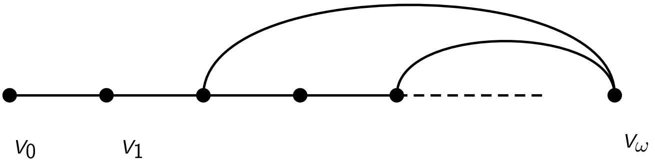

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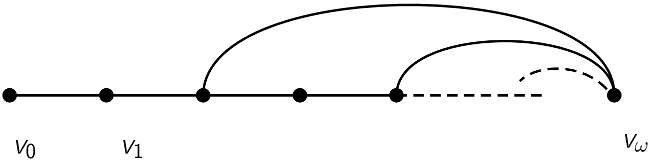

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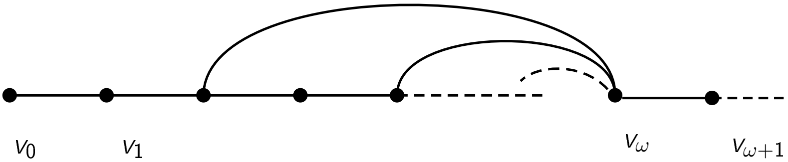

Arbitrary infinite complete graphs

A **path** is a graph P with w.o. $\prec$ so that any two points are connected by a finite $\prec$-monotone path.

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Is every 2-edge coloured infinite complete graph covered by two disjoint monochromatic paths of different colours?

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The ***chromatic number*** *of a graph G, denoted by $Chr(G)$, is the least (cardinal) number κ such that* ***the vertices of G can be covered by κ many independent sets****.*

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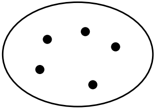

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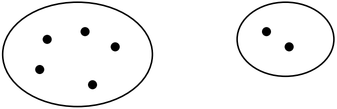

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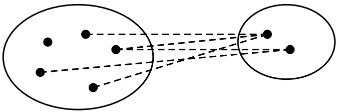

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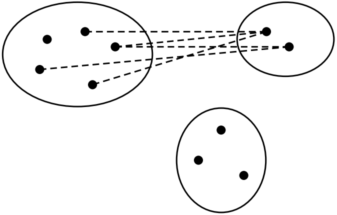

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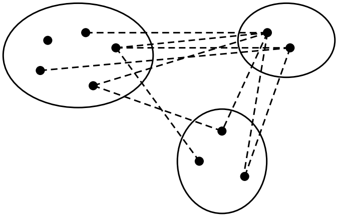

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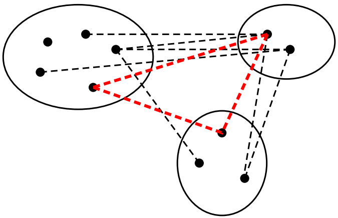

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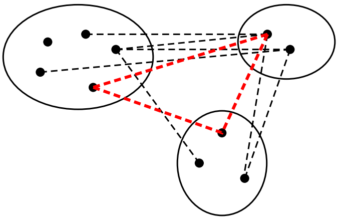

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The first results

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What graphs must occur as subgraphs of uncountably chromatic graphs?

- **Erdős-Rado, 1959:** There are $\triangle$-**free** graphs with size and **chromatic number** κ for each infinite κ.
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A set A in graph is **infinitely connected** iff every two points of A are connected by infinitely many disjoint paths in A.

[Erdős-Hajnal, 1966, 1985...] Does every graph G with uncountable chromatic number **contain infinitely connected subgraphs** with uncountable chromatic number?

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*There is a graph of **chromatic number** ω_1 and size continuum **without uncountable infinitely connected subgraphs**.*

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- consider the comparability graph of a non-special tree without uncountable chains.
- thin out the edges to have no uncountable infinitely connected subgraph:
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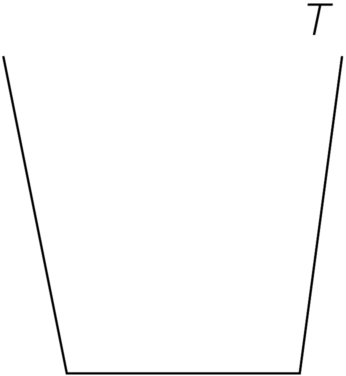

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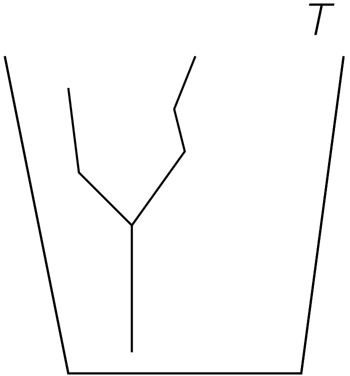

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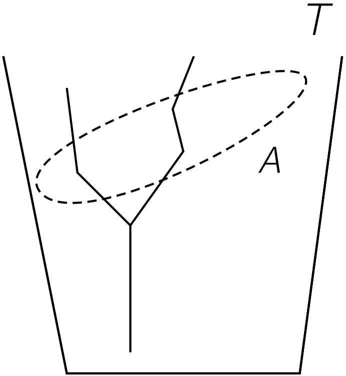

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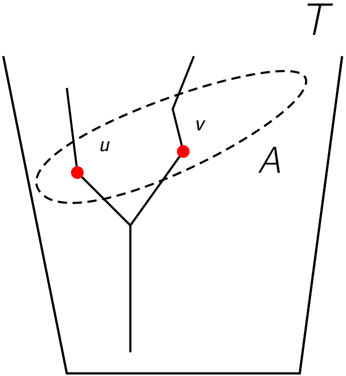

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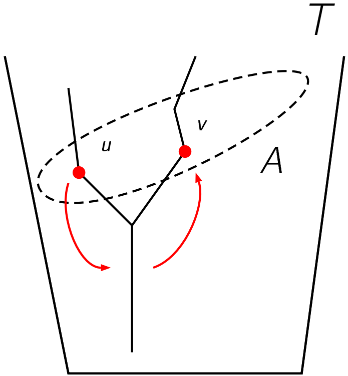

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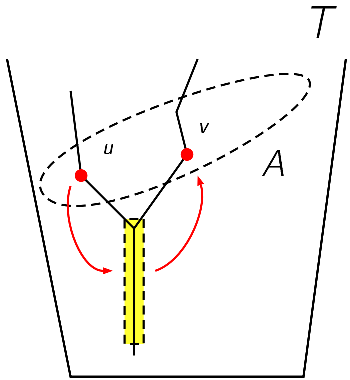

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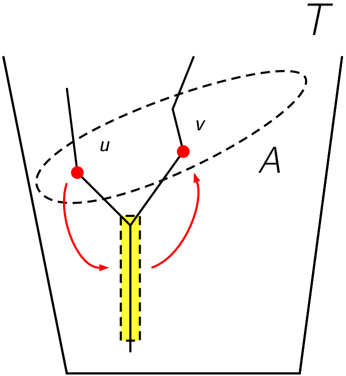

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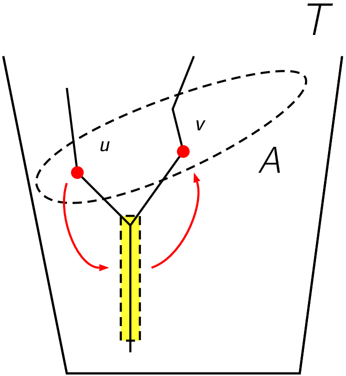

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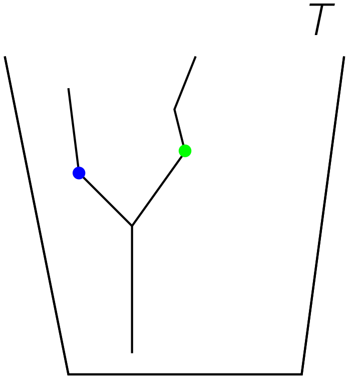

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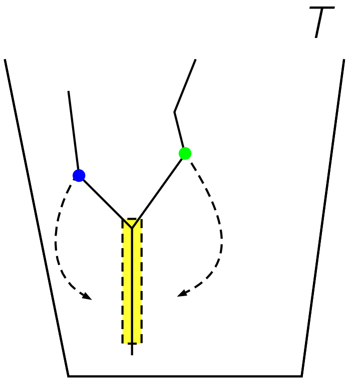

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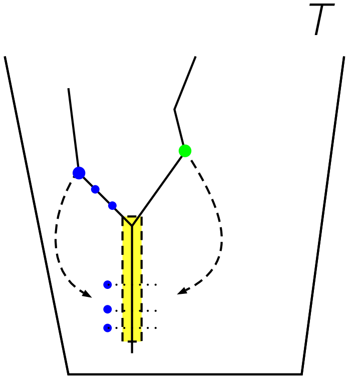

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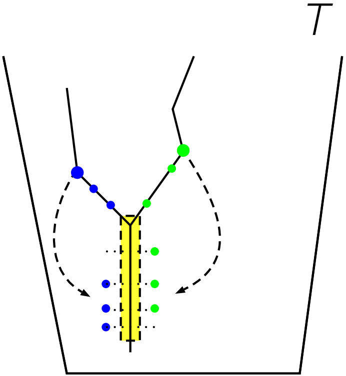

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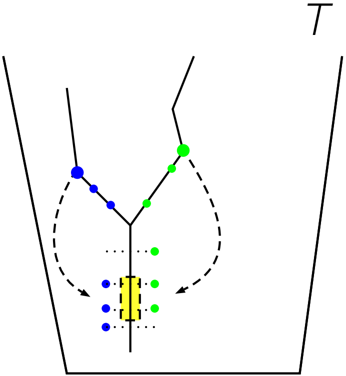

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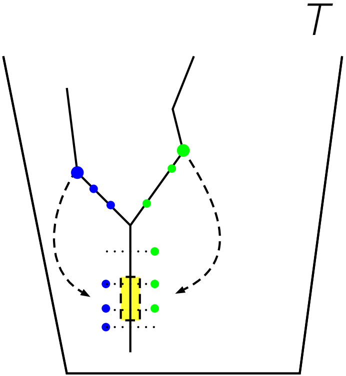

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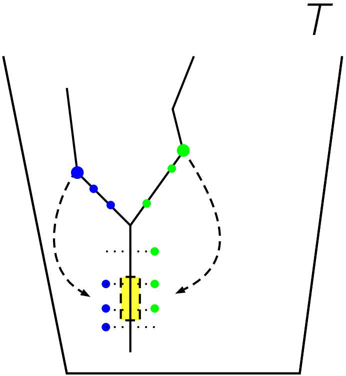

Thank you for your attention.

The infinite we do now, the finite will have to wait a little.

P. Erdős