Variations of (selective) separability

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properties stating that a space has a small dense set

- *d*-separable (property K_0):
  has a dense set which is the countable union of discrete subsets.
- *nwd*-separable:
  has a dense set which is the countable union of nowhere dense sets.

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On d-separable spaces

d-separable: has a σ-discrete dense set
nwd-separable: has a meager dense set

Examples:

- $D(2)^\kappa$ is d-separable as $\sigma(D(2)^\kappa)$ is σ-discrete.
- $s(X) < d(X)$ then X is not d-separable; in particular, L-spaces are not d-separable.

- (Arhangel'skii) Products of d-separable spaces are d-separable.
- (Juhász-Szentmiklóssy) $X^{d(X)}$ is d-separable.
- (Juhász-Szentmiklóssy) If X is compact, then X^ω is d-separable.

Example: let $X = \Sigma(2^{\omega_1})$ with the G_δ topology. Then X is not d-separable but every somewhere dense set contains a discrete subset of size $\mathfrak{c}$.

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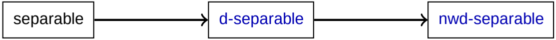
On nwd-separable spaces

Easy analogues and questions



- Any product of nwd-separable spaces is nwd-separable.
- Infinite (non trivial) products are always nwd-separable; in particular, X^ω is always *nwd*-separable.

Example: there is a compact nwd-separable space which is not d-separable: $X = \omega^* \times 2^\omega$.

- Is there a non-*nwd*-separable space with an *nwd*-separable square? (J. Moore) There is an L-space with a d-separable square!

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separable → d-separable → nwd-separable

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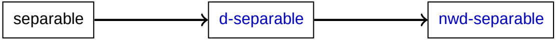

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Selection Principles

d-separable: has a σ-discrete dense set
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- a small dense set can be obtained by diagonalizing over a countable sequence of dense sets.
- a selective strengthening of properties:
  - D-separable:
    $\forall \{D_n\}_{n\in\omega} \subset \mathcal{D}$ $\exists$ discrete $F_n \subseteq D_n$ such that $\bigcup\{F_n : n \in \omega\} \in \mathcal{D}$;
  - NWD-separable:
    $\forall \{D_n\}_{n\in\omega} \subset \mathcal{D}$ $\exists$ nowhere dense $F_n \subseteq D_n$ such that $\bigcup\{F_n : n \in \omega\} \in \mathcal{D}$

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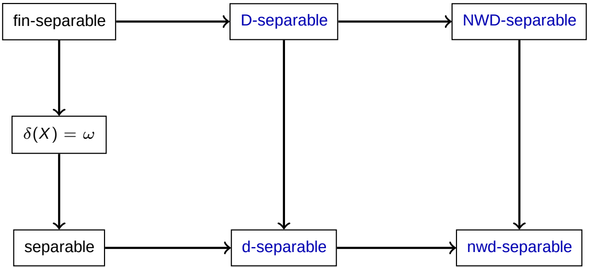
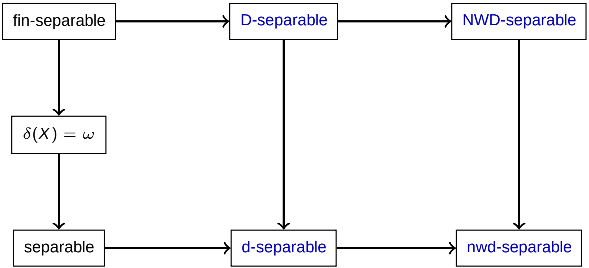


How can we separate these properties?



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Theorem

There is a countable dense subspace of $D(2)^{\mathfrak{c}}$ which is not NWD-separable.

- (J-S-Sz): there is a countable, dense subspace X of $D(2)^{\mathfrak{c}}$ s. t.
  - X can be partitioned into dense subspaces $\mathcal{D} = \{D_n : n \in \omega\}$
  - X is $\mathcal{D}$-forced, i.e. if $D \subset X$ is somewhere dense then $D_n \cap U \subseteq D$ for some $n \in \omega$ and non-empty open set U.
- Then X is not NWD-separable:
  - if $E_n \subset D_n$ is nowhere dense then $E = \bigcup_{n\in\omega} E_n$ is not dense, because it cannot contain any $D_n \cap U$.

Between selective and non-selective properties

A method of Juhász-L.Soukup-Szentmiklóssy

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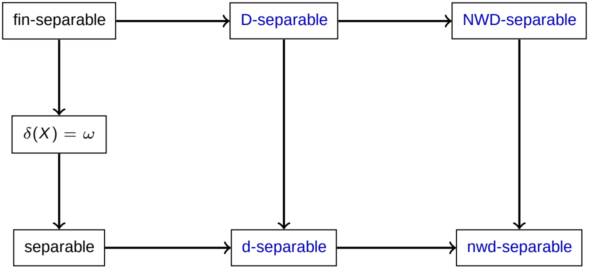
Selective separability and convergence



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- How about d-separability (nwd) and D-separability (NWD)?

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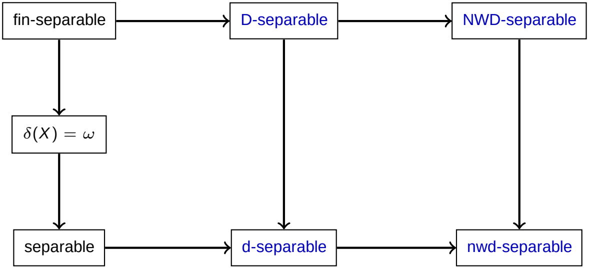

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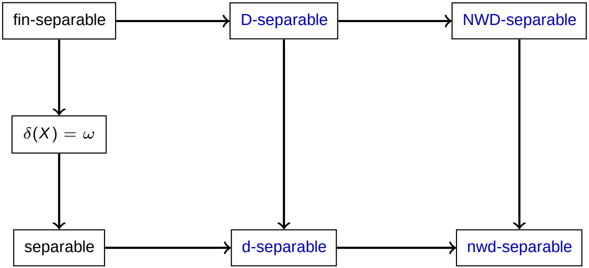

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Theorem

It is consistent that there is a first countable, 0-dimensional space X which is left-separated in type ω_1 such that X is NWD-separable but not d-separable.

The forcing:

- finite approximations of countable neighborhood bases then adding ω_2 Cohen reals;
- X is an L-space $\Rightarrow$ X is not d-separable;
- X is σ-nowhere dense and $MA_{\omega_1}(ctbl)$ holds $\Rightarrow$ X is NWD-separable.

- Are there any firstcountable ZFC examples? Compact?

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Every MN, nwd-separable space is D-separable.

- A space X is DDG iff for every $A \subseteq X$ and discrete $E \subset \overline{A}$ there is a discrete $D \subseteq A$ such that $E \subseteq \overline{D}$.

Theorem (Aurichi, Dias, Junqueira)

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Consider $\sigma(2^{\omega_1})$!

- $\sigma(2^{\omega_1})$ is d-separable (even σ-discrete).
- Is $\sigma(2^{\omega_1})$ D-separable?

Theorem

It is independentof ZFC whether $\sigma(2^{\omega_1})$ is DDG:

- *MA_{ω_1} implies that $\sigma(2^{\omega_1})$ is DDG hence D-separable.*
- *$\diamondsuit$ implies that $\sigma(2^{\omega_1})$ is not DDG.*

- Is it consistent that $\sigma(2^{\omega_1})$ is not D-separable?

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- $\sigma(2^{\omega_1})$ is d-separable (even σ-discrete).
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Various open problems

- Construct in ZFC a d-separable X such that X has no dense σ-discrete set of size $d(X)$!
- Is there a non D-separable space X (or even an L-space) such that X^2 is D-separable? There is an L-space with d-separable square!

Theorem (Bella-Matveev-Spadaro)

$X^{2^{d(X)}}$ is not D-separable for any Tychonoff space X.

- Conjecture: $X^{d(X)^+}$ is never D-separable! This is true for separable space!
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Thank you for your attention!