Partitioning bases of topological spaces

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The main question

Main Problem (B. Farkas)

Given a topological space X and a base $\mathbb{B}$ of X is there a partition of $\mathbb{B}$ into two bases?

- space X $\sim$ topological space with **no isolated points**.

Definition

A base $\mathbb{B}$ for a space X is ***resolvable*** *iff it can be decomposed into two bases. A space X is* ***base resolvable*** *if every base of X is resolvable.*

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Partition problems with topological flavour

- (Hewitt) Is there a partition of X into disjoint dense sets?
- (Baumgartner) There is a coloring $c : [\mathbb{Q}]^2 \to \omega$ such that $c''[A]^2 = \omega$ for every $A \subseteq \mathbb{Q}$ with $A \simeq \mathbb{Q}$.
- (Elekes, Mátrai, L. Soukup) There is an infinite fold cover $\mathcal{A}$ of $\mathbb{R}$ with translates of a single compact set such that there are no disjoint subcovers of $\mathbb{R}$ in $\mathcal{A}$.
- (Lindgren, Nyikos) Order properties of bases, Noetherian bases.

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Base resolvable classes

A base $\mathbb{B}$ is **resolvable** $\Leftrightarrow$ there is partition of $\mathbb{B}$ to disjoint bases.

(Stone) If a partial order $\mathbb{P}$ has **no maximal elements** then it admits a **partition to two cofinal sets**.

Observation

1. *Every neighborhood base can be partitioned to two neighborhood bases.*
2. *Every π-base can be partitioned to two π-bases.*
3. *Every base can be partitioned to a cover and a base.*

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Enough: having a σ-disjoint base.

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Non base resolvable examples

Theorem

*There is a (T_0) space X with a point countable, **non resolvable base** $\mathbb{B}$.*

Remark: X admits a compact, 1st countable topology τ and the non resolvable base $\mathbb{B}$ is a subfamily of τ.

Theorem (L. Soukup)

*It is consistent that there is a first countable, 0-dimensional, T_2 space which has a point countable, **non resolvable base**.*

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Access to paper: http://www.math.toronto.edu/∼ dsoukup/

- Is every **linearly ordered** space base resolvable?
- Is every T_3 (hereditarily) **separable** space base resolvable?
- Is every **homogeneous** space base resolvable?
- Is every **power of** $\mathbb{R}$ base resolvable?
- Is there a **non resolvable base** $\mathbb{B}$ for a topology on $\mathbb{R}$ such that every set in $\mathbb{B}$ is **Euclidean closed** (Borel)?

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